Optimal Peer Selection in a Free Market Peer-Resource Economy

Micah Adler	UMass, Amherst
Rakesh Kumar	Polytechnic University
Keith W. Ross	Polytechnic University
Dan Rubenstein	Columbia University
David Turner	CSU San Bernardino
David D. Yao	Columbia University

Introduction

- Problem of free-riders in P2P Systems.

- Need for a compelling incentive system.

Free Market
Peer Resource Economy

- Online Market Place – Peers buy and sell surplus resources.
- Sellers free to set prices and Buyers "shop" for the best deals.
- Example ?
- Focus: File sharing systems
  - Downloading & Streaming Apps.

Outline

- Pricing Model
- Problem of optimal peer selection
- Analysis : P2P Downloading
- Analysis : P2P Streaming

Pricing model : Goods

Goods which can be sold :

- Content
- Resources for content delivery

Pricing model : Content Delivery Resources

Factors effecting seller's price :

- Rate of transfer
- Duration of transfer

Pricing model (3)

- We consider pricing functions of the form

$$\text{price} = c_i(b_i)\ t_i$$

where b_i is the rate and t_i transfer duration

Outline

- Pricing Model for Resources.
- Problem of optimal peer selection
- Analysis : P2P Downloading
- Analysis : P2P Streaming

Problem of Optimal Peer Selection

- How to select a subset of sellers so that some optimization criteria are satisfied ?
- Typical optimization criteria could be ?
  - Minimize cost
  - Minimize latency
  - Maximize service reliability.

Outline

- Pricing Model for Resources.
- Problem of optimal peer selection
- Analysis : P2P Downloading
- Analysis : P2P Streaming

P2P Downloading: Customer's Perspective

- Customer's objective is to download the file in minimum time possible.
- Server's price increases with rate.
- Customer has a limited budget

Download Protocol

- Search in the overlay identifies, say, I seller peers offering the requested file.
- Each seller peer advertises its pricing function $c_i(b)$ and range of offered download rates $[l_i,u_i]$
- Customer chooses sellers and then downloads file chunks in parallel from the chosen sellers

P2P Downloading : Optimization Problem

Minimize:

delay of downloading file

Subject to:

(i) downloading cost is $\leq$ customer's budget =K
(ii) file, F bytes, is completely downloaded

Review of notation

- I servers have file
- $[l_i, u_i]$ = set of transfer rates available from server i
- b_i = chosen transfer rate from server i
- t_i = duration of chunk transfer from server i
- $c_i(b_i)\ t_i$ = server i's price
- K = buyer's budget
- F = file size

P2P Downloading : Optimization Problem (2)

$$\min \max \{t_1, t_2, \ldots, t_I\}$$

subject to

$$c_1(b_1)\, t_1 + c_2(b_2)\, t_2 + \ldots c_I(b_I)\, t_I \leq K$$

$$b_1 t_1 + b_2 t_2 + \ldots. + b_I t_I = F$$

$$l_i \leq b_i \leq u_i \qquad i = \{1, 2, \ldots, I\}$$

$$t_i \geq 0 \qquad i = \{1, 2, \ldots, I\}$$

P2P Downloading : Lemma

Lemma 1 : Suppose $c_i(b)/b$ is non-increasing. There exists an optimal solution with the property $b_i = u_i$ for all indices i such that $t_i>0$.

This lemma implies that we may set $b_i = u_i$ for all i, thus

w.l.o.g $\qquad c_i := c_i(u_i)/u_i$

P2P Downloading : Optimization Problem (2)

$$\min \max \{t_1, t_2, \ldots, t_I\}$$

subject to

$$c_1 t_1 + c_2 t_2 + \ldots c_I t_I \leq K$$

$$u_1 t_1 + u_2 t_2 + \ldots. + u_I t_I = F$$

$$t_i \geq 0 \qquad i = \{1, 2, \ldots, I\}$$

P2P Downloading : Solution(1)

Define $B_j = \sum_{k=1}^{j} u_k$ and $\beta_j = \sum_{k=1}^{j} c_i u_k$

- Case 1 : If customer peer has large enough budget

=> $\frac{K}{F} \geq \frac{\beta_I}{B_I}$ then buy all bandwidth offered

Solution: $$t_i = \frac{F}{\sum_{k=1}^{I} u_k}$$

P2P Downloading : Solution(1)

Define $B_j = \sum_{k=1}^{j} u_k$ and $\beta_j = \sum_{k=1}^{j} c_k u_k$

- Case 2 : If customer peer has a smaller budget

$$\frac{\beta_j}{B_j} \geq \frac{K}{F} \geq \frac{\beta_{j-1}}{B_{j-1}}, j \in [1, I]$$

Solution:

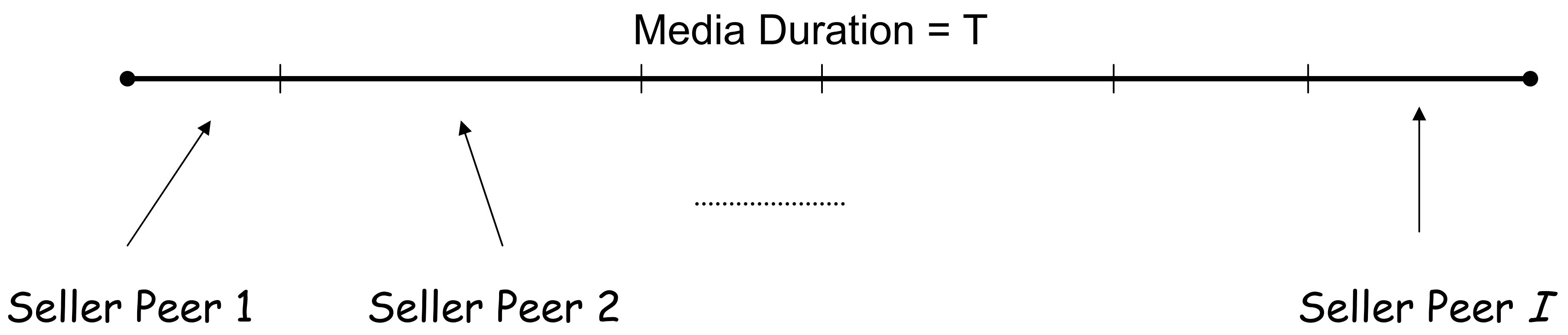
$$t_i = \begin{cases} \dfrac{Fc_j - K}{c_j B_j - \beta_j} & i \leq j-1 \\ F - B_{j-1} t_{j-1} & i = j \\ 0 & i > j \end{cases}$$

Outline

- Pricing Model for Resources.
- Problem of optimal peer selection
- Analysis : P2P Downloading
- Analysis : P2P Streaming

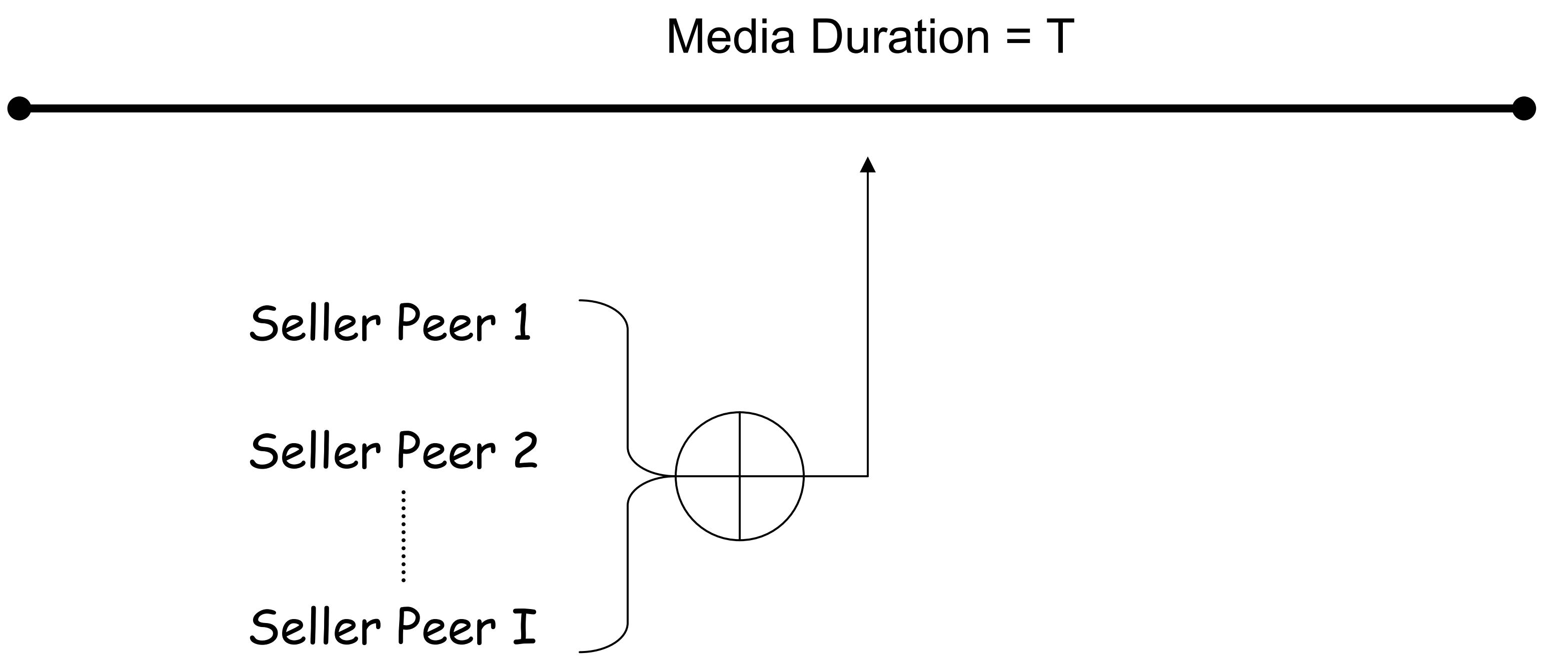
P2P Streaming: Possible Approaches(1)

- Time Segmentation
  - Video partitioned along the time axis.
  - Seller peers stream one segment each.
  - Customer peer buffers bytes untill scheduled play-out times.



P2P Streaming: Possible Approaches(2)

- Rate Segmentation
  - Each seller peer contributes bytes from each frame



P2P Streaming: Rate vs. Time Segmentation

Theorem : If $c_i(b)$ grow atleast linearly, then for any solution $\mathcal{S}$ that uses time segmentation, there is a solution $\mathcal{S}'$ using rate segmentation that has no larger cost.

P2P Streaming : Optimization Problem

Minimize:

cost of streaming

Subject to:

(i) continuous playback at rate r
(ii) even if one server peer goes down

P2P Streaming : Optimization Problem(2)

$$\min\ [c_1(b_1) + c_2(b_2) + \ldots + c_I(b_I)]*T$$

s.t.

$$\underbrace{\sum_{i=1}^{I}}_{i \neq j} b_i \geq r,\ j = 1,2,\ldots,I$$

P2P Streaming: Optimization Problem (3)

Alternatively

$$\min C(y), \quad y \in (0, r)$$

Where,

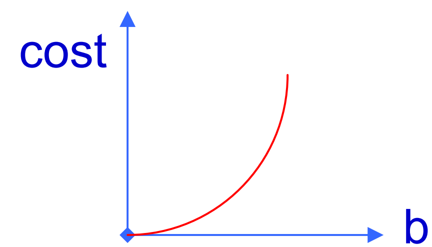
$$C(y) = \min\,[c_1(b_1) + c_2(b_2) + \ldots + c_I(b_I)]$$

$$b_1 + b_2 + \ldots + b_I \geq r + y$$

$$b_i \leq y,\ i = 1,2,\ldots I$$

P2P Streaming: convex costs

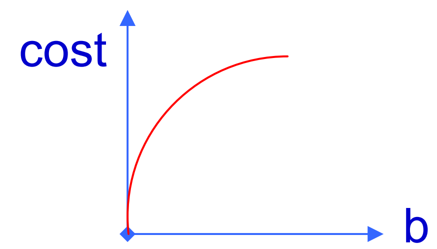
- **Theorem**: If all $c_i(b)$ are convex then $C(y)$ is also a convex function.



- **Solution Outline:**
  - For a given y, $C(y)$ is generated through marginal allocation.
  - Streaming problem is greedily solved by finding min $C(y)$, $0 \leq y \leq r$

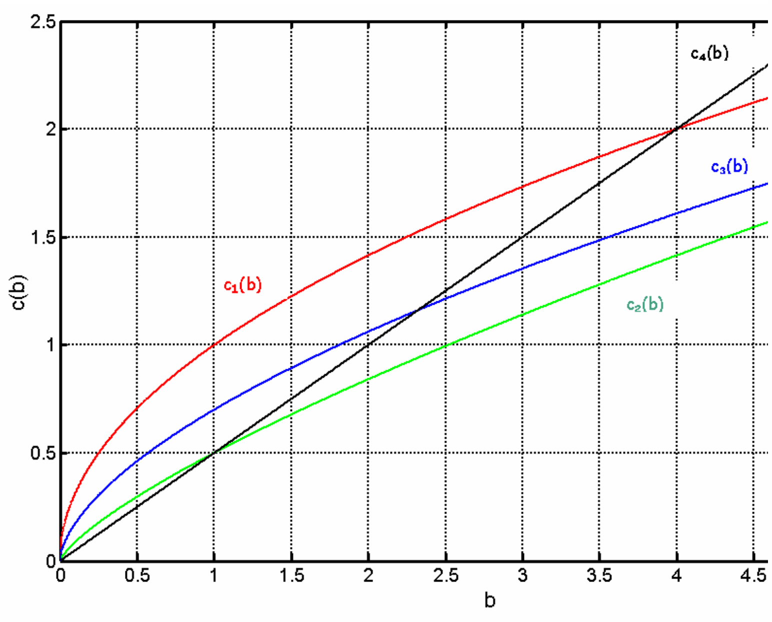
P2P Streaming: concave costs

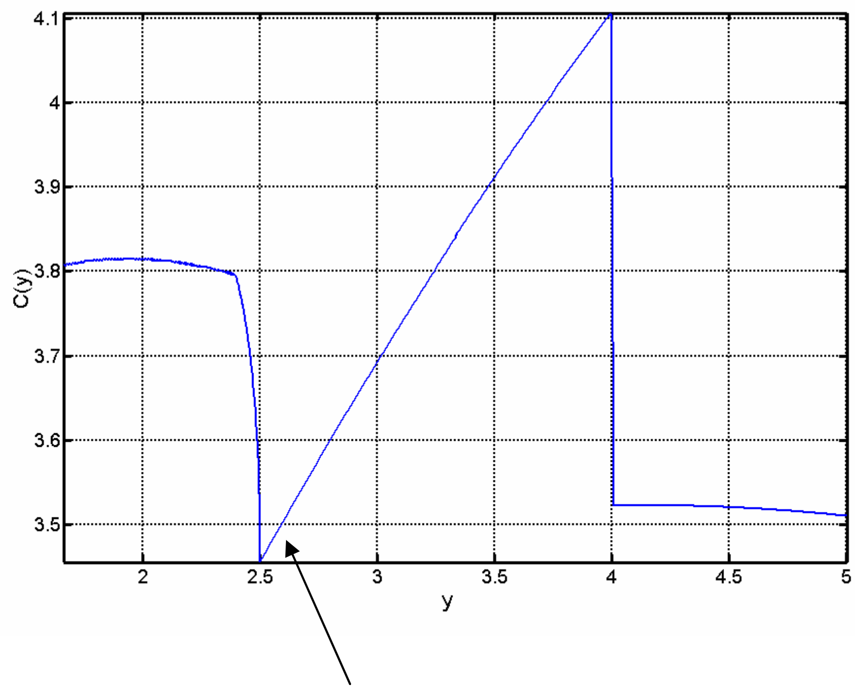
- **Theorem:** If all $c_i(b)$ concave, then we only need to consider the extreme points of feasible region.



- **Solution Outline:**
  - For a given y, solve $C(y)$ by computing minimum objective value at the extreme points.
  - Streaming problem is solved by line search to min $C(y)$, $0 \le y \le r$

P2P Streaming: concave costs - example





$$c_1(b) = \sqrt{b}$$
$$c_2(b) = 0.5b^{\frac{3}{4}}$$
$$c_3(b) = 0.7b^{\frac{3}{5}}$$
$$c_4(b) = 0.5b$$
$$r = 5$$

min C(y) = C(2.5)

Summary

- Use of free market resource economy model to provide incentives.
- Customer can make purchase decisions through an Agent
- Discussion for two applications : downloading and streaming.

Questions and Comments ?